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The Regularity of Almost-Commuting Partial Grothendieck-Springer Resolutions and Parabolic Analogs of Calogero-Moser Varieties

Consider the moment map $\mu\colon T^*(\mathfrak{p} \times \mathbb{C}^n) \to \mathfrak{p}^*$ for a parabolic subalgebra $\mathfrak{p}$ of $\mathfrak{gl}_n(\mathbb{C})$. We prove that the preimage of 0 under μ is a complete intersection when $\mathfrak{p}$ has finitely many P-orbits, where $P \subseteq \mathrm{GL}_n(\mathbb{C})$ is a parabolic subgroup such that $\mathrm{Lie}(P) = \mathfrak{p}$, and give an explicit description of the irreducible components. This allows us to study nearby fibers of μ as they are equidimensional, and one may also construct GIT quotients $\mu^{-1}(0)/\!\!/_\chi P$ by varying the stability condition χ. Finally, we study a variety analogous to the scheme studied by Wilson with connections to a Calogero-Moser phase space where only some of particles interact.

Keywords: Grothendieck-Springer resolution, moment map, complete intersection.

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