The Regularity of Almost-Commuting Partial Grothendieck-Springer Resolutions and Parabolic Analogs of Calogero-Moser Varieties

Consider the moment map \( \mu : T^*(p \times \mathbb{C}^n) \to p^* \) for a parabolic subalgebra \( p \) of \( \mathfrak{gl}_n(\mathbb{C}) \). We prove that the preimage of 0 under \( \mu \) is a complete intersection when \( p \) has finitely many \( P \)-orbits, where \( P \subseteq \text{GL}_n(\mathbb{C}) \) is a parabolic subgroup such that \( \text{Lie}(P) = p \), and give an explicit description of the irreducible components. This allows us to study nearby fibers of \( \mu \) as they are equidimensional, and one may also construct GIT quotients \( \mu^{-1}(0)/\chi P \) by varying the stability condition \( \chi \). Finally, we study a variety analogous to the scheme studied by Wilson with connections to a Calogero-Moser phase space where only some of particles interact.

**Keywords:** Grothendieck-Springer resolution, moment map, complete intersection.

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