J. Merker

Lab. de Mathématiques d'Orsay, CNRS, Université Paris-Saclay, Orsay, France
`joel.merker@universite-paris-saclay.fr`

A Lie-Theoretic Construction of Cartan-Moser Chains

Let $M^3 \subset \mathbb{C}^2$ be a real-analytic Levi nondegenerate hypersurface. In the literature, Cartan-Moser chains are detected from rather advanced considerations: either from the construction of a Cartan connection associated with the CR equivalence problem; or from the construction of a formal or converging Poincaré-Moser normal form.

This note provides an alternative direct elementary construction, based on the inspection of the Lie prolongations of 5 infinitesimal holomorphic automorphisms to the space of second order jets of CR-transversal curves. Within the 4-dimensional jet fiber, the orbits of these 5 prolonged fields happen to have a simple cubic 2-dimensional degenerate exceptional orbit, the *chain locus:*

$$\Sigma_0 := \left\{ (x_1, y_1, x_2, y_2) \in \mathbb{R}^4 \colon \ x_2 = -2x_1^2 y_1 - 2y_1^3, \ \ y_2 = 2x_1 y_1^2 + 2x_1^3 \right\}.$$

Using plain translations, we may capture all points by working *only at one point*, the origin, and computations become conceptually enlightening and simple.

Keywords: Lie prolongations of vector fields, Cauchy-Riemann manifolds, local biholomorphic equivalences, formal and convergent normal forms.

MSC: 32V40, 58K50, 34C20, 14R20; 53A55, 53B25, 14B10, 53-08, 53C30, 58K40, 58J70, 34C14, 58A30.