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On Lie Algebras from Polynomial Poisson Structures

We consider a polynomial Poisson algebra $\mathcal{P}$ on $\mathbb{R}^{2n}$ $(n \geq 1)$ that is to say $\mathcal{P}$ consists only of polynomials in $\mathbb{R}^{2n}$. We manage the conditions on $\mathcal{P}$ in order to have: every derivation of $\mathcal{P}$ is a differential operator of order one which takes its coefficients in $\mathcal{P}$. Otherwise, this result may not be true. More, we have an analogous result for the derived ideal $[\mathcal{P},\mathcal{P}]$ of $\mathcal{P}$. If $[\mathcal{P},\mathcal{P}] = \mathcal{P}$, derivations of the normalizer $\mathfrak{N}$ of $\mathcal{P}$ are sum of derivations of $\mathcal{P}$ and non-local derivations of $\mathfrak{N}$. Without this last hypothesis on $[\mathcal{P},\mathcal{P}]$, we can state a similar theorem about the normalizer of $[\mathcal{P},\mathcal{P}]$. The first Chevalley-Eilenberg cohomology of these sub-algebras are computed. Moreover, some results from polynomial Hamiltonian vector fields Lie algebras on $\mathbb{R}^{2n}$ has been found out. A special intention to Lie sub-algebras of the polynomial Poisson algebra $\mathbb{R}(x, y)$ on $\mathbb{R}^2$ in which the Jacobian conjecture holds is given. We give a definition on a sub-Lie algebra of $\mathbb{R}(x, y)$ verifying the Jacobian conjecture and find that if it is different to $\mathbb{R}(x, y)$, it verifies the Jacobian conjecture.

Keywords: Lie algebras, polynomial Poisson structure, Jacobian conjecture, cohomology of Chevalley-Eilenberg, differential operators, non-local derivations.

MSC: 17B66; 53B15, 17B56.