Journal of Lie Theory 30 (2020) 851–908

P. Paul
Dept. of Mathematics, Presidency University, Kolkata 700073, India
pampa.maths@presiuniv.ac.in

Geometric Cycles in Compact Riemannian Locally Symmetric Spaces of Type IV and Automorphic Representations of Complex Simple Lie Groups

Let G be a connected complex simple Lie group with maximal compact subgroup U. Let $\mathfrak{g}$ be the Lie algebra of G, and $X = G/U$ be the associated Riemannian globally symmetric space of type IV. We have constructed three types of arithmetic uniform lattices in G, say of type 1, type 2, and type 3 respectively. If $\mathfrak{g} \neq \mathfrak{b}_n$ $(n \geq 1)$, then for each $1 \leq i \leq 3$, there is an arithmetic uniform torsion-free lattice Γ in G which is commensurable with a lattice of type i such that the corresponding locally symmetric space $\Gamma\backslash X$ has some non-vanishing (in the cohomology level) geometric cycles, and the Poincaré duals of fundamental classes of such cycles are not represented by G-invariant differential forms on X. As a consequence, we are able to detect some automorphic representations of G, when $\mathfrak{g} = \delta_n$ $(n > 4)$, $\mathfrak{c}_n$ $(n \geq 6)$, or $\mathfrak{f}_4$. To prove these, we have simplified Kač's description of finite order automorphisms of $\mathfrak{g}$ with respect to a Chevalley basis of $\mathfrak{g}$. Also we have determined some orientation preserving group action on some subsymmetric spaces of X.

Keywords: Arithmetic lattice, automorphism of finite order of Lie algebra, orientation preserving isometry, geometric cycle, automorphic representation.

MSC: 22E40, 22E46, 22E15, 17B10, 17B40, 57S15.