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Spaces of Bounded Spherical Functions for Irreducible Nilpotent Gelfand Pairs: Part I

In prior work an orbit method, due to Pukanszky and Lipsman, was used to produce an injective mapping $\Psi \colon \Delta(K, N) \to \mathfrak{n}^*/K$ from the space of bounded K-spherical functions for a nilpotent Gelfand pair (K, N) into the space of K-orbits in the dual for the Lie algebra $\mathfrak{n}$ of N. We have conjectured that Ψ is a topological embedding. This has been proved for all pairs (K, N) with N a Heisenberg group. A nilpotent Gelfand pair (K, N) is said to be *irreducible* if K acts irreducibly on $\mathfrak{n}/[\mathfrak{n}, \mathfrak{n}]$. In this paper and its sequel we will prove that Ψ is an embedding for all such irreducible pairs. Our proof involves careful study of the non-Heisenberg entries in Vinberg's classification of irreducible nilpotent Gelfand pairs. Part I concerns generalities and six related families of examples from Vinberg's list in which the center for $\mathfrak{n}$ can have arbitrarily large dimension.

Keywords: Gelfand pairs, spherical functions, nilpotent Lie groups, orbit method.

MSC: 22E30, 43A90.