Irreducible Characters and Semisimple Coadjoint Orbits

When $G_\mathbb{R}$ is a real, linear algebraic group, the orbit method predicts that nearly all of the unitary dual of $G_\mathbb{R}$ consists of representations naturally associated to orbital parameters $(O, \Gamma)$. If $G_\mathbb{R}$ is a real, reductive group and $O$ is a semisimple coadjoint orbit, the corresponding unitary representation $\pi(O, \Gamma)$ may be constructed utilizing Vogan and Zuckerman’s cohomological induction together with Mackey’s real parabolic induction. In this article, we give a geometric character formula for such representations $\pi(O, \Gamma)$. Special cases of this formula were previously obtained by Harish-Chandra and Kirillov when $G_\mathbb{R}$ is compact and by Rossmann and Duflo when $\pi(O, \Gamma)$ is tempered.

Keywords: Semisimple orbit, coadjoint orbit, orbit method, Kirillov’s character formula, cohomological induction, parabolic induction, reductive group.

MSC: 22E46.