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Some Harmonic Analysis on Commutative Nilmanifolds

We consider a family of Gelfand pairs $(K \ltimes N, N)$ (in short (K, N)) where N is a two step nilpotent Lie group, and K is the group of orthogonal automorphisms of N . This family has a nice analytic property: almost all these 2-step nilpotent Lie group have square integrable representations. In these cases, following Moore-Wolf's theory, we find an explicit expression for the inversion formula of N , and as a consequence, we decompose the regular action of $K \ltimes N$ on $L^2(N)$. This explicit expression for the Fourier inversion formula of N , specialized to a class of commutative nilmanifolds described by J. Lauret, sharpens the analysis of J. A. Wolf in Section 14.5 in *Harmonic Analysis on Commutative Spaces* [Mathematical Surveys and Monographs 142, American Mathematical Society, Providence (2007)], and in *On the analytic structure of commutative nilmanifolds* [J. Geometric Analysis 26 (2016) 1011–1022], concerning the regular action of $K \ltimes N$ on $L^2(N)$. When N is the Heisenberg group, we obtain the decomposition of $L^2(N)$ under the action of $K \ltimes N$ for all K such that (K, N) is a Gelfand pair. Finally, we also give a parametrization for the generic spherical functions associated to the pair (K, N) , and we give an explicit expression for these functions in some cases.

Keywords: Gelfand pairs, inversion formula, nilpotent Lie group, regular representation.

MSC: 43A80, 22E25.