Journal of Lie Theory 30 (2020) 653–672

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Spherical Varieties over Large Fields

Let k_0 be a field of characteristic 0, k its algebraic closure, G a connected reductive group defined over k. Let $H \subset G$ be a spherical subgroup. We assume that k_0 is a large field, for example, k_0 is either the field $\mathbb{R}$ of real numbers or a p-adic field. Let G_0 be a quasi-split k_0-form of G. We show that if H has self-normalizing normalizer, and $\Gamma = \mathrm{Gal}\,(k/k_0)$ preserves the combinatorial invariants of G/H, then H is conjugate to a subgroup defined over k_0, and hence, the G-variety G/H admits a G_0-equivariant k_0-form. In the case when G_0 is not assumed to be quasi-split, we give a necessary and sufficient Galois-cohomological condition for the existence of a G_0-equivariant k_0-form of G/H.

Keywords: Equivariant form, inner form, algebraic group, spherical homogeneous space.

MSC: 20G15, 12G05, 14M17, 14G27, 14M27.