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# On the Areas of Level Sets in Compact Connected Sublattices of Three-Dimensional Euclidean Space

As is well-known the three-dimensional Euclidean space $\Re^3$, equipped with the order relation $(x_1, x_2, x_3) \leq (x_1', x_2', x_3')$ if $x_i \leq x_i'$ for $i = 1, 2, 3$, is a distributive, topological lattice. Let $L$ be a compact, connected sublattice of $\Re^3$. For $(x_1, x_2, x_3) \in L$ we define $\lambda(x_1, x_2, x_3) = x_1 + x_2 + x_3$ and for $r \in \Re$ we let $L_r = \{(x_1, x_2, x_3) \in L : \lambda(x_1, x_2, x_3) = r\}$. If $\mu_L(r)$ denotes the surface area of $L_r$, then we show that the function $r \mapsto \mu_L(r)$ is continuously differentiable, and that the value of $\mu_L'(r)$ can be computed in two different ways: Either as an integral of a certain function over the boundary of $L_r$, or as the value of the expression $\sqrt{3}\,(\lambda(\sup L_r) + \lambda(\inf L_r) - 2r)$.

**Keywords**: Level sets and rank functions, sublattices of R<sup>3</sup>, integral formulas.

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