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Locally Compact Groups with Compact Open Subgroups Having Open Chabauty Spaces

Let G be a locally compact group. We denote by $\mathcal{SUB}(G)$ the space of closed subgroups of G equipped with the Chabauty topology; this is a compact space. The topological space $\mathcal{SUB}(G)$ is called the Chabauty space of G. For a closed subgroup H of G the subspace $\{L \in \mathcal{SUB}(G) \mid L \subseteq H\}$ of $\mathcal{SUB}(G)$ is homeomorphic to the Chabauty space $\mathcal{SUB}(H)$ of H and so $\mathcal{SUB}(H)$ is a compact subspace of $\mathcal{SUB}(G)$. The paper discusses the scope of validity of an assertion having appeared recently in the book of Herfort-Hofmann-Russo about the openness of the subspace $\mathcal{SUB}(H)$ in $\mathcal{SUB}(G)$. We study the class \mathfrak{X} of locally compact groups G such that the subspace $\mathcal{SUB}(H)$ is open in $\mathcal{SUB}(G)$ for any compact open subgroup H of G. We show that a locally compact abelian group A is in \mathfrak{X} if and only if A contains a compact open subgroup U such that A/Uis a finite direct sum of subgroups each of which is either cyclic or is a Prüfer group isomorphic to $\mathbb{Z}(p^{\infty})$.

Keywords: Locally compact group, Chabauty topology, finitely cogenerated group, Pruefer group.

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