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The Rosenfeld Planes

We construct the Lie triples of the exceptional symmetric spaces with dimensions 16, 32, 64, 128 and isometry groups of Dynkin types F_4, E_6, E_7, E_8. We start with a certain representation for the spin group $Spin_{k+l}$ by 2×2-matrices over $\mathbb{K} \otimes \mathbb{L}$ where $\mathbb{K}, \mathbb{L}$ are normed real division algebras with dimensions $k, l \in \{1, 2, 4, 8\}$. The Lie triple is $(\mathbb{K} \otimes \mathbb{L})^2$ with this representation of $Spin_{k+l}$ as isotropy representation, extended by certain scalars in $\mathbb{K} \otimes \mathbb{L}$. Six of these ten representations are shown to be isotropy representations of classical symmetric spaces. This observation greatly simplifies the check of Jacobi identity and the computation of the root decomposition. We call the corresponding symmetric spaces (generalized) Rosenfeld planes. They contain half dimensional subspaces with Lie triple $(\mathbb{K} \otimes \mathbb{L})^1$, so called Rosenfeld lines, which are shown to be certain real Grassmannians.

Keywords: Division algebras, Spin groups, exceptional Lie groups, exceptional symmetric spaces.

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