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**Invariant Differential Operators on Spherical Homogeneous Spaces
with Overgroups**

We investigate the structure of the ring $\mathbb{D}_G(X)$ of G -invariant differential operators on a reductive spherical homogeneous space $X = G/H$ with an overgroup \tilde{G} . We consider three natural subalgebras of $\mathbb{D}_G(X)$ which are polynomial algebras with explicit generators, namely the subalgebra $\mathbb{D}_{\tilde{G}}(X)$ of \tilde{G} -invariant differential operators on X and two other subalgebras coming from the centers of the enveloping algebras of \mathfrak{g} and \mathfrak{k} , where K is a maximal proper subgroup of G containing H . We show that in most cases $\mathbb{D}_G(X)$ is generated by any two of these three subalgebras, and analyze when this may fail. Moreover, we find explicit relations among the generators for each possible triple (\tilde{G}, G, H) , and describe *transfer maps* connecting eigenvalues for $\mathbb{D}_{\tilde{G}}(X)$ and for the center of the enveloping algebra of $\mathfrak{g}_{\mathbb{C}}$.

Keywords: Branching law, spherical variety, real spherical variety, symmetric space, invariant differential operator, enveloping algebra.

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