Journal of Lie Theory 29 (2019) 511–558

J. Frahm
Department Mathematik, FAU Erlangen-Nürnberg, Cauerstr. 11, 91058 Erlangen, Germany
frahm@math.fau.de

C. Weiske
Department Mathematik, FAU Erlangen-Nürnberg, Cauerstr. 11, 91058 Erlangen, Germany
weiske@math.fau.de

Symmetry Breaking Operators for Line Bundles over Real Projective Spaces

The space of smooth sections of an equivariant line bundle over the real projective space $\mathbb{R}\mathrm{P}^n$ forms a natural representation of the group $\mathrm{GL}(n+1,\mathbb{R})$. We explicitly construct and classify all intertwining operators between such representations of $\mathrm{GL}(n+1,\mathbb{R})$ and its subgroup $\mathrm{GL}(n,\mathbb{R})$, intertwining for the subgroup. Intertwining operators of this form are called symmetry breaking operators, and they describe the occurrence of a representation of $\mathrm{GL}(n,\mathbb{R})$ inside the restriction of a representation of $\mathrm{GL}(n+1,\mathbb{R})$. In this way, our results contribute to the study of branching problems for the real reductive pair $(\mathrm{GL}(n+1,\mathbb{R}),\mathrm{GL}(n,\mathbb{R}))$.
The analogous classification is carried out for intertwining operators between algebraic sections of line bundles, where the Lie group action of $\mathrm{GL}(n,\mathbb{R})$ is replaced by the action of its Lie algebra $\mathfrak{gl}(n,\mathbb{R})$, and it turns out that all intertwining operators arise as restrictions of operators between smooth sections.

Keywords: Symmetry breaking operators, real projective spaces, general linear group, intertwining operators, Harish-Chandra modules, principal series.

MSC: 22E46; 17B15, 46F12