Journal of Lie Theory 29 (2019) 457–471

H. Abels
Fakultä für Mathematik, Universität Bielefeld, 33501 Bielefeld, Germany
abels@math.uni-bielefeld.de

G. A. Noskov
Sobolev Institute of Mathematics, Pevtsova 13, 644099 Omsk, Russia
g.noskov@googlemail.com

The Topological Generating Rank of Solvable Lie Groups

We define the topological generating rank $d(G)$ of a connected Lie group G as the minimal number of elements of G needed to generate a dense subgroup of G. We answer the following question posed by K. H. Hofmann and S. A. Morris [see: *Finitely generated connected locally compact groups*, J. Lie Theory (formerly Sem. Sophus Lie) 2(2) (1992) 123–134]: What is the topological generating rank of a connected solvable Lie group? If G is solvable we can reduce the question to the case that G is metabelian. We can furthermore reduce to the case that the natural representation of $Q{:=}G^{ab}{:=}G/\overline{G'}$ on $A := \overline{G'}$ is semisimple. Then $d(G)$ is the maximum of the following two numbers: $d(Q)$ and one plus the maximum of the multiplicities of the non-trivial isotypic components of the $\mathbb{R}Q$-module A.

Keywords: Lie group, solvable, nilpotent, metabelian, topological generators, generating rank.

MSC: 20E25