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The Pro-Lie Group Aspect of Weakly Complete Algebras and Weakly Complete Group Hopf Algebras

A weakly complete vector space over $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$ is isomorphic to $\mathbb{K}^X$ for some set X algebraically and topologically. The significance of this type of topological vector spaces is illustrated by the fact that the underlying vector space of the Lie algebra of any pro-Lie group is weakly complete. In this study, weakly complete real or complex associative algebras are studied because they are necessarily projective limits of finite dimensional algebras. The group of units A^{-1} of a weakly complete algebra A is a pro-Lie group with the associated topological Lie algebra $A_{\rm Lie}$ of A as Lie algebra and the globally defined exponential function $\exp\colon A \to A^{-1}$ as the exponential function of A^{-1}. With each topological group, a weakly complete group algebra $\mathbb{K}[G]$ is associated functorially so that the functor $G \mapsto \mathbb{K}[G]$ is left adjoint to $A \mapsto A^{-1}$. The group algebra $\mathbb{K}[G]$ is a weakly complete Hopf algebra. If G is compact, the $\mathbb{R}[G]$ contains G as the set of grouplike elements. The category of all real Hopf algebras A with a compact group of grouplike elements whose linear span is dense in A is shown to be equivalent to the category of compact groups. The group algebra $A = \mathbb{R}[G]$ of a compact group G contains a copy of the Lie algebra $\mathfrak{L}(G)$ in $A_{\rm Lie}$; it also contains a copy of the Radon measure algebra $M(G, \mathbb{R})$. The dual of the group algebra $\mathbb{R}[G]$ is the Hopf algebra $\mathcal{R}(G, \mathbb{R})$ of representative functions of G. The rather straightforward duality between vector spaces and weakly complete vector spaces thus becomes the basis of a duality $\mathcal{R}(G, \mathbb{R}) \leftrightarrow \mathbb{R}[G]$ and thus yields a new aspect of Tannaka duality.

Keywords: Pro-Lie group, weakly complete vector space, weakly complete algebra, group algebra, Hopf algebra, measure algebra, compact group, Tannaka duality.

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