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A. Baklouti
Faculté des Sciences de Sfax, Dép. de Mathématiques, Sfax 3038, Tunisia
ali.baklouti@fss.usf.tn

H. Fujiwara
Fac. of Science and Technology, University of Kinki, Iizuka 820-8555, Japan
fujiwara6913@yahoo.co.jp

J. Ludwig
Institut Elie Cartan, Université de Lorraine, 57000 Metz, France
jean.ludwig@univ-lorraine.fr

The Polynomial Conjecture for Restrictions of Some Nilpotent Lie Groups Representations

Let G be a connected and simply connected nilpotent Lie group, K an analytic subgroup of G and π an irreducible unitary representation of G whose coadjoint orbit of G is denoted by $\Omega(\pi)$. Let $\mathcal{U}(\mathfrak{g})$ be the enveloping algebra of $\mathfrak{g}_{\mathbb{C}}$, $\mathfrak{g}$ designating the Lie algebra of G. We consider the algebra $(\mathcal{U}(\mathfrak{g})/\ker\pi)^K$ of the K-invariant elements of $\mathcal{U}(\mathfrak{g})/\ker\pi$. It turns out that this algebra is commutative if and only if the restriction $\pi|_K$ of π to K has finite multiplicities (cf. A. Baklouti and H. Fujiwara, *Commutativité des opérateurs différentiels sur l'espace des représentations restreintes d'un groupe de Lie nilpotent*, J. Math. Pures Appl. 83 (2004) 137–161). In this article we suppose this eventuality and we study the polynomial conjecture asserting that our algebra is isomorphic to the algebra $\mathbb{C}[\Omega(\pi)]^K$ of the K-invariant polynomial functions on $\Omega(\pi)$. We give a proof of the conjecture in the case where $\Omega(\pi)$ admits a normal polarization of G and in the case where K is abelian. This problem was partially tackled previously by A. Baklouti, H. Fujiwara, J. Ludwig, *Analysis of restrictions of unitary representations of a nilpotent Lie group*, Bull. Sci. Math. 129 (2005) 187–209.

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