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Galois Groups and Group Actions on Lie Algebras

If $\mathfrak{g} \subseteq \mathfrak{h}$ is an extension of Lie algebras over a field k such that $\dim_k(\mathfrak{g}) = n$ and $\dim_k(\mathfrak{h}) = n + m$, then the Galois group $\mathrm{Gal}\,(\mathfrak{h}/\mathfrak{g})$ is explicitly described as a subgroup of the canonical semidirect product of groups $\mathrm{GL}(m,\, k) \rtimes \mathrm{M}_{n\times m}(k)$. An Artin type theorem for Lie algebras is proved: if a group G whose order is invertible in k acts as automorphisms on a Lie algebra $\mathfrak{h}$, then $\mathfrak{h}$ is isomorphic to a skew crossed product $\mathfrak{h}^G \,\#^{\bullet}\, V$, where $\mathfrak{h}^G$ is the subalgebra of invariants and V is the kernel of the Reynolds operator. The Galois group $\mathrm{Gal}\,(\mathfrak{h}/\mathfrak{h}^G)$ is also computed, highlighting the difference from the classical Galois theory of fields where the corresponding group is G. The counterpart for Lie algebras of Hilbert's Theorem 90 is proved and based on it the structure of Lie algebras $\mathfrak{h}$ having a certain type of action of a finite cyclic group is described. Radical extensions of finite dimensional Lie algebras are introduced and it is shown that their Galois group is solvable. Several applications and examples are provided.

Keywords: Groups acting on Lie algebras, Galois groups, Artin's theorem, Hilbert's theorem 90.

MSC: 17B05, 17B40.