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Do n-Lie Algebras Have Universal Enveloping Algebras?

The aim of this paper is to investigate in which sense, for $n \geq 3$, n-Lie algebras admit universal enveloping algebras. There have been some attempts at a construction (see A. S. Dzhumadil'daev, Representations of vector product n-Lie algebras, Comm. Algebra 32 (2004) 3315–3326, and D. Bălibanu and J. van de Leur, Irreducible highest weight representations of the simple n-Lie algebra, Transform. Groups 17 (2012) 593–613), but after analysing those we come to the conclusion that they cannot be valid in general. We give counterexamples and sufficient conditions.

We then study the problem in its full generality, showing that universality is incompatible with the wish that the category of modules over a given n-Lie algebra L is equivalent to the category of modules over the associated algebra $U(L)$. Indeed, an *associated algebra functor* U: n-Lie$\rightarrow$ Alg$_{\mathbb{K}}$ inducing such an equivalence does exist, but this kind of functor never admits a right adjoint.

We close the paper by introducing a (co)homology theory based on the associated algebra functor U.

Keywords: n-Lie, n-Leibniz, universal enveloping algebra.

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