Journal of Lie Theory 27 (2017) 193–215

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On the Construction of Simply Connected Solvable Lie Groups

Let $\omega_{\mathfrak{g}}$ be a Lie algebra valued differential 1-form on a manifold M satisfying the structure equations $d\omega_{\mathfrak{g}} + \frac{1}{2}\omega_{\mathfrak{g}} \wedge \omega_{\mathfrak{g}} = 0$, where $\mathfrak{g}$ is a solvable real Lie algebra. We show that the problem of finding a smooth map $\rho\colon M \to G$, where G is an n-dimensional solvable real Lie group with Lie algebra $\mathfrak{g}$ and left invariant Maurer-Cartan form τ, such that $\rho^*\tau = \omega_{\mathfrak{g}}$ can be solved by quadratures and the matrix exponential. In the process, we give a closed form formula for the vector fields in Lie's third theorem for solvable Lie algebras. A further application produces the multiplication map for a simply connected n-dimensional solvable Lie group using only the matrix exponential and n quadratures. Applications to finding first integrals for completely integrable Pfaffian systems with solvable symmetry algebras are also given.

Keywords: Solvable Lie algebras, solvable Lie groups, Lie's third theorem, first integrals.

MSC: 22E25; 58A15, 58J70, 34A26