

© 2016 Heldermann Verlag  
Journal of Lie Theory 26 (2016) 269–291

**A. Deitmar**

Mathematisches Institut, Auf der Morgenstelle 10, 72076 Tübingen, Germany  
deitmar@uni-tuebingen.de

**G. van Dijk**

Mathematical Institute, Niels Bohrweg 1, 2333 CA Leiden, The Netherlands  
dijk@math.leidenuniv.nl

**Trace Class Groups**

A representation  $\pi$  of a locally compact group  $G$  is called *trace class*, if for every test function  $f$  the induced operator  $\pi(f)$  is a trace class operator. The group  $G$  is called *trace class*, if every  $\pi \in \widehat{G}$  is trace class. In this paper we give a survey of what is known about trace class groups and ask for a simple criterion to decide whether a given group is trace class. We show that trace class groups are type I and give a criterion for semi-direct products to be trace class and show that a representation  $\pi$  is trace class if and only if  $\pi \otimes \pi'$  can be realized in the space of distributions.

**Keywords:** Trace class operator, type I group, unitary representation.

**MSC:** 22D10, 11F72, 22D30, 43A65