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Divergence and q -Divergence in Depth 2

The Kashiwara-Vergne Lie algebra \mathfrak{krv} encodes symmetries of the Kashiwara-Vergne problem on the properties of the Campbell-Hausdorff series. It is conjectured that $\mathfrak{krv} \cong \mathbb{K}t \oplus \mathfrak{gr}t_1$, where t is a generator of degree 1 and $\mathfrak{gr}t_1$ is the Grothendieck-Teichmüller Lie algebra. In this paper, we prove this conjecture in depth 2. The main tools in the proof are the divergence cocycle and the representation theory of the dihedral group D_{12} . Our calculation is similar to the calculation by Zagier of the graded dimensions of the double shuffle Lie algebra in depth 2.

In analogy to the divergence cocycle, we define the super-divergence and q -divergence cocycles (here $q^l = 1$) on Lie subalgebras of $\mathfrak{gr}t_1$ which consist of elements with weight divisible by l . We show that in depth 2 these cocycles have no kernel. This result is in sharp contrast with the fact that the divergence cocycle vanishes on $[\mathfrak{gr}t_1, \mathfrak{gr}t_1]$.

Keywords: Kashiwara-Vergne conjecture, divergence cocycle, Grothendieck-Teichmüller Lie algebra.

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