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Sur les champs de vecteurs invariants sur l'espace tangent d'un espace symétrique réductif

Let G be a real reductive and connected Lie group and σ an involution of G. Let H denote the identity component of the group of fixed points of σ, $\mathfrak{g}$ the Lie algebra of G and $\mathfrak{q}$ the -1 eigenspace of σ in $\mathfrak{g}$. The group H acts naturally on $\mathfrak{q}$ via the adjoint representation. Let $C^\infty(\mathfrak{q})^H$ denote the algebra of H-invariant smooth functions on $\mathfrak{q}$, and $\mathfrak{X}(\mathfrak{q})^H$ the space of H-invariant smooth vector fields on $\mathfrak{q}$. Any vector field $X \in \mathfrak{X}(\mathfrak{q})^H$ defines naturally a derivation D_X of the algebra $C^\infty(\mathfrak{q})^H$. We prove that the image of the map $X \mapsto D_X$ is the set of derivations of the algebra $C^\infty(\mathfrak{q})^H$ preserving the ideal $\Phi C^\infty(\mathfrak{q})^H$ of $C^\infty(\mathfrak{q})^H$, where Φ is a discriminant function on $\mathfrak{q}$.

Keywords: Lie Group, symmetric space, invariant vector field, Taylor expansion.

MSC: 17B20, 22F30, 22E30