Journal of Lie Theory 25 (2015) 009–036

M. Voit
Fakultät Mathematik, Technische Universität, Vogelpothsweg 87, 44221 Dortmund, Germany
michael.voit@math.tu-dortmund.de

Product Formulas for a Two-Parameter Family of Heckman-Opdam Hypergeometric Functions of Type BC

We present explicit product formulas for a continuous two-parameter family of Heckman-Opdam hypergeometric functions of type BC on Weyl chambers $C_q \subset \mathbb{R}^q$ of type B. These formulas are related to continuous one-parameter families of probability-preserving convolution structures on $C_q \times \mathbb{R}$. These convolutions on $C_q \times \mathbb{R}$ are constructed via product formulas for the spherical functions of the symmetric spaces $U(p,q)/(U(p) \times SU(q))$ and associated double coset convolutions on $C_q \times \mathbb{T}$ with the torus $\mathbb{T}$. We shall obtain positive product formulas for a restricted parameter set only, while the associated convolutions are always norm-decreasing.
Our paper is related to recent positive product formulas of Rösler for three series of Heckman-Opdam hypergeometric functions of type BC as well as to classical product formulas for Jacobi functions of Koornwinder and Trimeche for rank $q = 1$.

Keywords: Hypergeometric functions associated with root systems, Heckman-Opdam theory, hypergroups, product formulas, Grassmann manifolds, spherical functions, signed hypergroups, Haar measure.

MSC: 33C67, 43A90, 43A62, 33C80