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Existence of Lattices on General H-Type Groups

Let $\mathcal{N}$ be a two step nilpotent Lie algebra endowed with non-degenerate scalar product $\langle \cdot, \cdot \rangle$ and let $\mathcal{N} = V \oplus_{\perp} Z$, where Z is the center of the Lie algebra and V its orthogonal complement with respect to the scalar product. We prove that if $(V, \langle \cdot, \cdot \rangle_V)$ is the Clifford module for the Clifford algebra $\mathrm{Cl}(Z, \langle \cdot, \cdot \rangle_Z)$ such that the homomorphism $J \colon \mathrm{Cl}(Z, \langle \cdot, \cdot \rangle_Z) \to \mathrm{End}(V)$ is skew symmetric with respect to the scalar product $\langle \cdot, \cdot \rangle_V$, or in other words the Lie algebra $\mathcal{N}$ satisfies conditions of general H-type Lie algebras [see P. Ciatti, Scalar products on Clifford modules and pseudo-H-type Lie algebras, Math. Nachr. 202 (2009) 44–68; and: M. Godoy Molina, A. Korolko and I. Markina, Sub-semi-Riemannian geometry of general H-type groups, Bull. Sci. Math. 137 (2013) 805–833], then there is a basis with respect to which the structural constants of the Lie algebra $\mathcal{N}$ are all ± 1 or 0.

Keywords: Clifford module, nilpotent two step algebra, lattice, general H-type algebras.

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