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Lifting Automorphisms of Quotients of Adjoint Representations

Let $\mathfrak{g}_i$ be a simple complex Lie algebra, $1 \leq i \leq d$, and let $G = G_1 \times \cdots \times G_d$ be the corresponding adjoint group. Consider the G-module $V = \oplus r_i \mathfrak{g}_i$ where $r_i \in \mathbb{N}$ for all i. We say that V is *large* if all $r_i \geq 2$ and $r_i \geq 3$ if G_i has rank 1. In "Quotients, automorphisms and differential operators", http://arxiv.org/abs/1201.6369 (2012), we showed that when V is large any algebraic automorphism ψ of the quotient $Z := V/\!\!/G$ lifts to an algebraic mapping $\Psi\colon V \to V$ which sends the fiber over z to the fiber over $\psi(z)$, $z \in Z$. (Most cases were already handled in J. Kuttler, Lifting automorphisms of generalized adjoint quotients, Transformation Groups **16** (2011) 1115–1135.) We also showed that one can choose a biholomorphic lift Ψ such that $\Psi(gv) = \sigma(g)\Psi(v)$, $g \in G$, $v \in V$, where σ is an automorphism of G. This leaves open the following questions: Can one lift holomorphic automorphisms of Z? Which automorphisms lift if V is not large? We answer the first question in the affirmative and also answer the second question. Part of the proof involves establishing the following result for V large: Any algebraic differential operator of order k on Z lifts to a G-invariant algebraic differential operator of order k on V. We also consider the analogues of the questions above for actions of compact Lie groups.

Keywords: Differential operators, automorphisms, quotients, adjoint representation.

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