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Kac-Moody Lie Algebras Graded by Kac-Moody Root Systems

We look to gradations of Kac-Moody Lie algebras by Kac-Moody root systems with finite dimensional weight spaces. We extend, to general Kac-Moody Lie algebras, the notion of C-admissible pair as introduced by H. Rubenthaler and J. Nervi for semi-simple and affine Lie algebras. If $\mathfrak{g}$ is a Kac-Moody Lie algebra (with Dynkin diagram indexed by I) and (I, J) is such a C-admissible pair, we construct a C-admissible subalgebra $\mathfrak{g}^J$, which is a Kac-Moody Lie algebra of the same type as $\mathfrak{g}$, and whose root system Σ grades finitely the Lie algebra $\mathfrak{g}$. For an admissible quotient $\rho : I \to \overline{I}$ we build also a Kac-Moody subalgebra $\mathfrak{g}^\rho$ which grades finitely the Lie algebra $\mathfrak{g}$. If $\mathfrak{g}$ is affine or hyperbolic, we prove that the classification of the gradations of $\mathfrak{g}$ is equivalent to those of the C-admissible pairs and of the admissible quotients. For general Kac-Moody Lie algebras of indefinite type, the situation may be more complicated; it is (less precisely) described by the concept of generalized C-admissible pairs.

Keywords: Kac-Moody algebra, C-admissible pair, gradation.

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