Journal of Lie Theory 23 (2013) 1011–1022

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Projections of Orbital Measures, Gelfand-Tsetlin Polytopes, and Splines

The unitary group $U(N)$ acts by conjugations on the space $\mathcal{H}(N)$ of $N \times N$ Hermitian matrices, and every orbit of this action carries a unique invariant probability measure called an orbital measure. Consider the projection of the space $\mathcal{H}(N)$ onto the real line assigning to an Hermitian matrix its $(1,1)$-entry. Under this projection, the density of the pushforward of a generic orbital measure is a spline function with N knots. This fact was pointed out by Andrei Okounkov in 1996, and the goal of the paper is to propose a multidimensional generalization. Namely, it turns out that if instead of the $(1,1)$-entry we cut out the upper left matrix corner of arbitrary size $K \times K$, where $K = 2, \dots, N-1$, then the pushforward of a generic orbital measure is still computable: its density is given by a $K \times K$ determinant composed from one-dimensional splines. The result can also be reformulated in terms of projections of the Gelfand-Tsetlin polytopes.

Keywords: Orbital measure, Gelfand-Tsetlin polytope, B-spline, Harish-Chandra-Itzykson-Zuber integral.

MSC: 22E30 41A15