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Graded Oscillator Generalizations of the Classical Theorem on Harmonic Polynomials

Classical harmonic analysis says that the spaces of homogeneous harmonic polynomials (solutions of Laplace equation) are irreducible modules of the corresponding orthogonal Lie group (algebra) and the whole polynomial algebra is a free module over the invariant polynomials generated by harmonic polynomials. Algebraically, this gives an $(sl(2,\mathbb{R}), o(n,\mathbb{R}))$ Howe duality. In this paper, we study two-parameter oscillator variations of the above theorem associated with noncanonical oscillator representations of $o(n,\mathbb{C})$. We find the condition when the homogeneous solution spaces of the variated Laplace equation are irreducible modules of $o(n,\mathbb{C})$ and the homogeneous subspaces are direct sums of the images of these solution subspaces under the powers of the dual differential operator. This establishes an $(sl(2,\mathbb{C}), o(n,\mathbb{C}))$ Howe duality on some homogeneous subspaces. In generic case, the obtained irreducible $o(n,\mathbb{C})$-modules are infinite-dimensional non-unitary modules without highest-weight vectors. When both parameters are equal to the maximal allowed value, we obtain explicit irreducible $(\mathcal{G},\mathcal{K})$-modules for $o(n,\mathbb{C})$.

Keywords: Orthogonal Lie algebra, harmonic polynomial, oscillator representation, irreducible module, invariant operator.

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