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**Interior Regularity for Degenerate Elliptic Equations with Drift on Homogeneous Groups**

Let  $G$  be a homogeneous group and let  $X_0, X_1, X_2, \dots, X_{p_0}$  be left invariant real vector fields on  $G$  satisfying Hörmander's rank condition. Assume that  $X_1, X_2, \dots, X_{p_0}$  are homogeneous of degree one and  $X_0$  is homogeneous of degree two. In this paper, we study the following equation with drift:

$$Lu \equiv \sum_{i,j=1}^{p_0} X_i(a_{ij}(x)X_j u) + a_0 X_0 u = \sum_{j=1}^{p_0} X_j F_j(x),$$

where  $a_{ij}(x)$  are real valued, bounded measurable functions defined in a domain  $\Omega \subset G$ ,  $a_{ij}(x) = a_{ji}(x)$ , satisfying the uniform ellipticity condition in  $\mathbb{R}^{p_0}$  and  $a_0 \in \mathbb{R} \setminus \{0\}$ . Moreover, the coefficients  $a_{ij}$  belong to the class  $VMO$  (Vanishing Mean Oscillation) with respect to the subelliptic metric induced by the vector fields  $X_0, X_1, X_2, \dots, X_{p_0}$ . We derive local  $L^p$  estimates for second order derivatives and Hölder estimates by establishing the representation formulas and higher order integrability of weak solutions to the above equation.

**Keywords:** Homogeneous group, interior regularity, vector fields.

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