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A Characterization of the Unitary Highest Weight Modules by Euclidean Jordan Algebras

Let $\mathfrak{co}(J)$ be the conformal algebra of a simple Euclidean Jordan algebra J. We show that a (non-trivial) unitary highest weight $\mathfrak{co}(J)$-module has the smallest positive Gelfand-Kirillov dimension if and only if a certain quadratic relation is satisfied in the universal enveloping algebra $U(\mathfrak{co}(J)_{\mathbb{C}})$. In particular, we find an quadratic element in $U(\mathfrak{co}(J)_{\mathbb{C}})$. A prime ideal in $U(\mathfrak{co}(J)_{\mathbb{C}})$ equals the Joseph ideal if and only if it contains this quadratic element.

Keywords: Euclidean Jordan algebras, unitary highest weight module, quadratic relation, Joseph Ideal.

MSC: 22E47, 17B10, 17C99