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Journal of Lie Theory 23 (2013) 589–606

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**Capelli Elements for the Algebra  $\mathfrak{g}_2$**

M. Itoh and T. Umeda [On Central Elements in the Universal Enveloping Algebras of the Orthogonal Lie Algebras, *Compositio Mathematica* 127 (2001) 333–359] constructed central elements in the universal enveloping algebra  $U(\mathfrak{o}_N)$ , named Capelli elements, as sums of squares of noncommutative Pfaffians of some matrices, whose entries belong to  $\mathfrak{o}_N$ . However for exceptional algebras there are no construction of this type. In the present paper we construct central elements in  $U(\mathfrak{g}_2)$  as sums of squares of Pfaffians of some matrices, whose elements belong to  $\mathfrak{g}_2$ . For  $U(\mathfrak{g}_2)$ , as in the case  $U(\mathfrak{o}_N)$ , we give characterization of these central elements in terms of their vanishing properties. Also for  $U(\mathfrak{g}_2)$  an explicit relations between constructed central elements and higher Casimir elements defined by D. P. Zhelobenko [Compact Lie groups and their representations, Amer. Math. Soc., Providence, R.I. (1973)] are obtained.

**Keywords:** Central elements, universal enveloping algebra, pfaffian.

**MSC:** 17B25, 16S30