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V. M. Petrogradsky
Faculty of Mathematics, Ulyanovsk State University, Leo Tolstoy 42, Ulyanovsk 432970, Russia
petrogradsky@rambler.ru

I. P. Shestakov
Instituto de Mathemática e Estatística, Universidade de Sa Paulo, Caixa postal 66281, CEP 05315-970, Sa Paulo, Brazil
shestak@ime.usp.br

On Properties of the Fibonacci Restricted Lie Algebra

Let $R = K[t_i | i \geq 0]/(t_i^p | i \geq 0)$ be the truncated polynomial ring, where K is a field of characteristic 2. Let $\partial_i = \frac{\partial}{\partial t_i}$, $i \geq 1$, denote the respective derivations. Consider the operators

$$v_1 = \partial_1 + t_0(\partial_2 + t_1(\partial_3 + t_2(\partial_4 + t_3(\partial_5 + t_4(\partial_6 + \cdots)))));$$

$$v_2 = \partial_2 + t_1(\partial_3 + t_2(\partial_4 + t_3(\partial_5 + t_4(\partial_6 + \cdots)))) .$$

Let $\mathcal{L} = \mathrm{Lie}(v_1, v_2)$ and $\mathbb{L} = \mathrm{Lie}_p(v_1, v_2) \subset \mathrm{Der}\, R$ be the Lie algebra and the restricted Lie algebra generated by these derivations, respectively. These algebras were introduced by the first author and called Fibonacci Lie algebras. It was established that $\mathbb{L}$ has polynomial growth and a nil p-mapping. The latter property is a natural analogue of periodicity of Grigorchuk and Gupta-Sidki groups. We also proved that $\mathbb{L}$, the associative algebra generated by these derivations $\mathbb{A} = \mathrm{Alg}(v_1, v_2) \subset \mathrm{End}(R)$, and the augmentation ideal of the restricted enveloping algebra $u_0(\mathbb{L})$ are direct sums of two locally nilpotent subalgebras.
The goal of the present paper is to study Fibonacci Lie algebras in more details. We give a clear basis for the algebras $\mathbb{L}$ and $\mathcal{L}$. We find functional equations and recurrence formulas for generating functions of $\mathbb{L}$ and $\mathcal{L}$, also we find explicit formulas for these functions. We determine the center, terms of the lower central series, values of regular growth functions, and terms of the derived series of $\mathcal{L}$. We observed before that $\mathbb{L}$ is not just infinite dimensional. Now we introduce one more restricted Lie algebra $\mathbb{G} = \mathrm{Lie}_p(\partial_1, v_2)$ and prove that it is just infinite dimensional. Finally, we formulate open problems.

Keywords: Growth, self-similar algebras, nil-algebras, graded algebras, restricted Lie algebras, Lie algebras of differential operators, Fibonacci numbers.

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