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**Reflections on  $S^3$  and Quaternionic Möbius Transformations**

Let  $S^3$  be the set of unit quaternions, let  $\mathcal{H}$  be the algebra of quaternions, and let  $\mathcal{H}^*$  be the space of pure quaternions. It is an elementary fact that  $S^3$  and  $\mathcal{H}^* \cup \{\infty\}$  are homeomorphic spaces by a stereographic projection. We show that a reflection in  $S^3$  induces a linear fractional transformation on  $\mathcal{H}^* \cup \{\infty\}$  that is defined by a matrix in a symplectic group  $Sp(2)$ . In addition, we identify the left eigenvalues of such a matrix, and show the subgroup  $G$  generated by these matrices satisfies  $G/(\pm I_2) \simeq O(4)$ .

**Keywords:** Moebius transformation, quaternion.

**MSC:** 51B10, 15B33