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Homomorphisms of Generalized Verma Modules, BGG Parabolic Category $\mathcal{O}^{\mathfrak{p}}$ and Juhl's Conjecture

Let $\mathcal{M}_\lambda(\mathfrak{g}, \mathfrak{p})$, $\mathcal{M}_\mu(\mathfrak{g}', \mathfrak{p}')$ be the generalized Verma modules for $\mathfrak{g} = \mathrm{so}(p+1, q+1), \mathfrak{g}' = \mathrm{so}(p, q+1)$ induced from characters λ, μ of the standard maximal parabolic (conformal) subalgebras $\mathfrak{p}$, $\mathfrak{p}' = \mathfrak{g}' \cap \mathfrak{p}$. Motivated by questions about the existence of invariant differential operators in conformal geometry, we explain, reformulate and prove an extended version of Juhl's conjecture on the structure of $\mathcal{U}(\mathfrak{g}')$-homomorphisms of generalized Verma modules from $\mathcal{M}_\lambda(\mathfrak{g}', \mathfrak{p}')$ to $\mathcal{M}_\mu(\mathfrak{g}, \mathfrak{p})$. The answer has a natural formulation as a branching problem in the BGG parabolic category $\mathcal{O}^{\mathfrak{p}'}$ rather than the set of generalized Verma modules alone.

Keywords: Branching rules, generalized Verma modules, BGG parabolic category O^p, Juhl's conjectures.

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