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## Some Transitive Linear Actions of Real Simple Lie Groups

In a recent paper of M. Moskowitz and R. Sacksteder [An extension of the Minkowski-Hlawka theorem, Mathematika 56 (2010) 203-216], essential use was made of the fact that in its natural linear action the real symplectic group, $\mathrm{Sp}(n,\mathbb{R})$, acts transitively on $\mathbb{R}^{2n}\setminus\{0\}$ (similarly for the theorem of Hlawka itself, $\mathrm{SL}(n,\mathbb{R})$ acts transitively on $\mathbb{R}^{n}\setminus\{0\}$). This raises the natural question as to whether there are *proper connected* Lie subgroups of either of these groups which also act transitively on $\mathbb{R}^{2n}\setminus\{0\}$, (resp. $\mathbb{R}^{n}\setminus\{0\}$). Here we determine all the minimal ones. These are $\mathrm{Sp}(n,\mathbb{R})\subseteq\mathrm{SL}(2n,\mathbb{R})$ and $\mathrm{SL}(n,\mathbb{C})\subseteq\mathrm{SL}(2n,\mathbb{R})$ acting on $\mathbb{R}^{2n}\setminus\{0\}$; on $\mathbb{R}^{4n}\setminus\{0\}$, they are $\mathrm{Sp}(2n,\mathbb{R})\subseteq\mathrm{SL}(4n,\mathbb{R})$ and $\mathrm{SL}(n,\mathbb{H})(=\mathrm{SU}^*(2n))\subseteq\mathrm{SL}(4n,\mathbb{R})$.

**Keywords**: Transitive linear action, reductive group, actions of compact groups on spheres, special linear and real symplectic groups.

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