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Journal of Lie Theory 22 (2012) 137–153

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**Invariant Distributions on Non-Distinguished Nilpotent Orbits with
Application to the Gelfand Property of $(GL_{2n}(\mathbb{R}), Sp_{2n}(\mathbb{R}))$**

We study invariant distributions on the tangent space to a symmetric space. We prove that an invariant distribution with the property that both its support and the support of its Fourier transform are contained in the set of non-distinguished nilpotent orbits, must vanish. We deduce, using recent developments in the theory of invariant distributions on symmetric spaces, that the symmetric pair $(GL_{2n}(\mathbb{R}), Sp_{2n}(\mathbb{R}))$ is a Gelfand pair. More precisely, we show that for any irreducible smooth admissible Fréchet representation (π, E) of $GL_{2n}(\mathbb{R})$ the space of continuous functionals $Hom_{Sp_{2n}(\mathbb{R})}(E, \mathbb{C})$ is at most one dimensional. Such a result was previously proven for p -adic fields by M. J. Heumos and S. Rallis [Symplectic-Whittaker models for Gl_n , Pacific J. Math. 146 (1990) 247–279], and for \mathbb{C} by the second author [$(GL_{2n}(\mathbb{C}), Sp_{2n}(\mathbb{C}))$ is a Gelfand pair, arXiv:0805.2625, math.RT].

Keywords: Symmetric pair, Gelfand pair, symplectic group, non-distinguished orbits, multiplicity one, invariant distribution, co-isotropic subvariety.

MSC: 20G05, 22E45, 20C99, 46F10