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**Conformally Invariant Systems of Differential Equations on Flag Manifolds for  $G_2$  and their  $K$ -Finite Solutions**

Let  $G$  be the connected, split, linear real Lie group of type  $G_2$  and  $K$  a maximal compact subgroup of  $G$ . Several conformally invariant systems of partial differential equations on line bundles  $\mathcal{L} \rightarrow G/Q$ , where  $Q$  is a maximal real parabolic subgroup of  $G$ , are considered. In each case, the space of  $K$ -finite solutions to the system is determined explicitly, and this is then used to obtain some information about the space of smooth solutions. The conformal invariance of the systems implies that each of these solution spaces is a representation of  $G$ , and it is shown that they are irreducible as such.

**Keywords:** Conformally invariant system, explicit solutions, real flag manifold, hypergeometric vectors.

**MSC:** 22E47; 35R03, 35C11