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The Smoothness of Orbital Measures on Exceptional Lie Groups and Algebras

Suppose that G is a compact, connected, simple, exceptional Lie group with Lie algebra \mathfrak{g} . We determine the sharp minimal exponent k_0 , which depends on G or \mathfrak{g} , such that the convolution of any k_0 continuous, G -invariant measures is absolutely continuous with respect to Haar measure. The exponent k_0 is also the minimal integer such that any k_0 -fold product of conjugacy classes in G or k_0 -fold sum of adjoint orbits in \mathfrak{g} has non-empty interior. Unlike in the classical case, the answer can be less than the rank of G or \mathfrak{g} .

We also establish a dichotomy for orbital measures μ , supported on non-trivial conjugacy classes or adjoint orbits of minimal non-zero dimension: for each k , either $\mu^k \in L^2$ or μ^k is singular with respect to Haar measure.

Keywords: Compact Lie group, compact Lie algebra, orbital measure, orbit, conjugacy class.

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