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The Commutator Subalgebra and Schur Multiplier of a Pair of Nilpotent Lie Algebras

Let (L, N) be a pair of finite dimensional nilpotent Lie algebras, in which N is an ideal in L . In the present article, we prove that if the factor Lie algebras L/N and $N/Z(L, N)$ are of dimensions m and n , respectively, then the commutator subalgebra $[L, N]$ is of dimension at most $\frac{1}{2}n(n + 2m - 1)$, and also determine when $\dim([L, N]) = \frac{1}{2}n(n + 2m - 1)$. In addition, we introduce the notion of the Schur multiplier $\mathcal{M}(L, N)$ of an arbitrary pair (L, N) of Lie algebras, and show that if N admits a complement K in L with $\dim(N) = n$ and $\dim(K) = m$, then the dimension of $\mathcal{M}(L, N)$ is bounded above by $\frac{1}{2}n(n + 2m - 1)$. In this case, we characterize the pairs (L, N) for which $\dim(\mathcal{M}(L, N))$ is either $\frac{1}{2}n(n + 2m - 1)$ or $\frac{1}{2}n(n + 2m - 1) - 1$.

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