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The Tame Algebra

The tame subgroup I_t of the Iwahori subgroup I and the tame Hecke algebra $H_t{=}C_c(I_t\backslash G/I_t)$ are introduced. It is shown that the tame algebra has a presentation by means of generators and relations, similar to that of the Iwahori-Hecke algebra $H = C_c(I\backslash G/I)$. From this it is deduced that each of the generators of the tame algebra is invertible. This has an application concerning an irreducible admissible representation π of an unramified reductive p-adic group G: π has a nonzero vector fixed by the tame group, and the Iwahori subgroup I acts on this vector by a character χ, iff π is a constituent of the representation induced from a character of the minimal parabolic subgroup, denoted χ_A, which extends χ. The proof is an extension to the tame context of an unpublished argument of Bernstein, which he used to prove the following. An irreducible admissible representation π of a quasisplit reductive p-adic group has a nonzero Iwahori-fixed vector iff it is a constituent of a representation induced from an unramified character of the minimal parabolic subgroup. The invertibility of each generator of H_t is finally used to give a Bernstein-type presentation of H_t, also by means of generators and relations, as an extension of an algebra with generators indexed by the finite Weyl group, by a finite index maximal commutative subalgebra, reflecting more naturally the structure of G and its maximally split torus.

Keywords: Tame algebra, Iwahori-Hecke Algebra, induced representation.

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