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On the Multiplication Groups of Three-Dimensional Topological Loops

We clarify the structure of nilpotent Lie groups which are multiplication groups of 3-dimensional simply connected topological loops and prove that non-solvable Lie groups acting minimally on 3-dimensional manifolds cannot be the multiplication group of 3-dimensional topological loops. Among the nilpotent Lie groups for all filiform groups $\mathcal{F}_{n+2}$ and $\mathcal{F}_{m+2}$ with $n, m > 1$, the direct product $\mathcal{F}_{n+2} \times \mathbb{R}$ and the direct product $\mathcal{F}_{n+2} \times_Z \mathcal{F}_{m+2}$ with amalgamated center Z occur as the multiplication group of 3-dimensional topological loops. To obtain this result we classify all 3-dimensional simply connected topological loops having a 4-dimensional nilpotent Lie group as the group topologically generated by the left translations.

Keywords: Multiplication group of loops, topological transformation group, filiform Lie group.

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