

K. H. Hofmann

Fachbereich Mathematik, Technische Universität, Schlossgartenstr. 7, 64289 Darmstadt, Germany
hofmann@mathematik.tu-darmstadt.de

S. A. Morris

Graduate School of Information Technology and Mathematical Sciences, University of Ballarat, P. O. 663, Ballarat, Vic. 3353, Australia
morris.sidney@gmail.com

The Structure of Almost Connected Pro-Lie Groups

Recalling that a topological group G is said to be almost connected if the quotient group G/G_0 is compact, where G_0 is the connected component of the identity, we prove that for an almost connected pro-Lie group G , there exists a compact zero-dimensional, that is, profinite, subgroup D of G such that $G = G_0 D$. Further for such a group G , there are sets I, J , a compact connected semisimple group S , and a compact connected abelian group A such that G and $\mathbb{R}^I \times (\mathbb{Z}/2\mathbb{Z})^J \times S \times A$ are homeomorphic. En route to this powerful structure theorem it is shown that the compact open topology makes the automorphism group $\text{Aut } \mathfrak{g}$ of a semisimple pro-Lie algebra $\mathfrak{g}$ a topological group in which the identity component $(\text{Aut } \mathfrak{g})_0$ is exactly the group $\text{Inn } \mathfrak{g}$ of inner automorphisms. In this situation, $\text{Inn}(G)$ has a totally disconnected semidirect complement Δ such that $\text{Aut } \mathfrak{g} = (\text{Inn } \mathfrak{g})\Delta$ and $\text{Aut } \mathfrak{g}/\text{Inn } \mathfrak{g} \cong \Delta$ as topological groups. The group $\text{Inn } \mathfrak{g}$ is a product of a family of connected simple centerfree Lie groups.

Keywords: Pro-Lie group, almost connected, maximal compact subgroup, conjugacy of subgroups, automorphism groups.

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