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Journal of Lie Theory 21 (2011) 079–099

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**Double Flag Varieties for a Symmetric Pair and Finiteness of Orbits**

Let  $G$  be a reductive algebraic group over the complex number field, and  $K = G^\theta$  be the fixed points of an involutive automorphism  $\theta$  of  $G$  so that  $(G, K)$  is a symmetric pair.

We take parabolic subgroups  $P$  and  $Q$  of  $G$  and  $K$  respectively and consider a product of partial flag varieties  $G/P$  and  $K/Q$  with diagonal  $K$ -action. The double flag variety  $G/P \times K/Q$  thus obtained is said to be *of finite type* if there are finitely many  $K$ -orbits on it. A triple flag variety  $G/P^1 \times G/P^2 \times G/P^3$  is a special case of our double flag varieties, and there are many interesting works on the triple flag varieties.

In this paper, we study double flag varieties  $G/P \times K/Q$  of finite type. We give efficient criterion under which the double flag variety is of finite type. The finiteness of orbits is strongly related to spherical actions of  $G$  or  $K$ . For example, we show a partial flag variety  $G/P$  is  $K$ -spherical if a product of partial flag varieties  $G/P \times G/\theta(P)$  is  $G$ -spherical. We also give many examples of the double flag varieties of finite type, and for type AIII, we give a classification when  $P = B$  is a Borel subgroup of  $G$ .

**Keywords:** Symmetric pair, flag variety, spherical action.

**MSC:** 14M15; 53C35, 14M17