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Sheets of Symmetric Lie Algebras and Slodowy Slices

Let θ be an involution of the finite dimensional reductive Lie algebra $\mathfrak{g}$ and $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ be the associated Cartan decomposition. Denote by $K \subset G$ the connected subgroup having $\mathfrak{k}$ as Lie algebra. The K-module $\mathfrak{p}$ is the union of the subsets $\mathfrak{p}^{(m)} := \{x \mid \dim K.x = m\}$, $m \in \mathbb{N}$, and the K-sheets of $(\mathfrak{g}, \theta)$ are the irreducible components of the $\mathfrak{p}^{(m)}$. The sheets can be, in turn, written as a union of so-called Jordan K-classes. We introduce conditions in order to describe the sheets and Jordan classes in terms of Slodowy slices. When $\mathfrak{g}$ is of classical type, the K-sheets are shown to be smooth; if $\mathfrak{g} = \mathfrak{gl}_N$ a complete description of sheets and Jordan classes is then obtained.

Keywords: Semisimple Lie algebra, symmetric Lie algebra, sheet, Jordan class, Slodowy slice, nilpotent orbit, root system.

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