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Journal of Lie Theory 20 (2010) 617–641

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Dirac Induction for Harish-Chandra Modules

Let G be a connected real reductive Lie group with Cartan involution Θ , such that $K = G^\Theta$ is a maximal compact subgroup of G , and such that G and K have equal rank. Let \mathfrak{g} be the complexified Lie algebra of G . We introduce new notions of Dirac cohomology and homology of a Harish-Chandra module X for the pair (\mathfrak{g}, K) . If X is unitary or finite-dimensional, then these new notions both coincide with the version of Dirac cohomology introduced by Vogan and further studied by Huang-Pandžić and others. The new notions have certain advantages. Notably, if X is irreducible and has nonzero Dirac cohomology (respectively homology), then X is uniquely determined by its Dirac cohomology (respectively homology). Furthermore, one can define adjoint functors that we call Dirac induction functors. We study basic properties of these functors and we calculate their result explicitly in some examples.

Keywords: Lie group, Lie algebra, representation, Harish-Chandra module, Dirac operator, Dirac cohomology, Dirac induction.

MSC: 22E47, 22E46