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Bounded Simple ($\mathfrak{g}$, sl(2))-Modules for rk $\mathfrak{g}$=2

This paper is a continuation of our work *On bounded generalized Harish-Chandra modules*, preprint (2009), math.jacobs-university.de/penkov, in which we prove some general results about simple $(\mathfrak{g}, \mathfrak{k})$-modules with bounded $\mathfrak{k}$-multiplicities (or bounded simple $(\mathfrak{g}, \mathfrak{k})$-modules). In the absence of a classification of bounded simple $(\mathfrak{g}, \mathfrak{k})$-modules in general, it is important to understand some special cases as best as possible. Here we consider the case $\mathfrak{k} = \mathrm{sl}(2)$. It turns out that in order for an infinite-dimensional bounded simple $(\mathfrak{g}, \mathrm{sl}(2))$-module to exist, $\mathfrak{g}$ must have rank 2, and, up to conjugation, there are five possible embeddings $\mathrm{sl}(2) \to \mathfrak{g}$ which yield infinite-dimensional bounded simple $(\mathfrak{g}, \mathrm{sl}(2))$-modules.
Our main result is a detailed description of the bounded simple $(\mathfrak{g}, \mathrm{sl}(2))$-modules in all five cases. When $\mathfrak{g} \simeq \mathrm{sl}(2) \oplus \mathrm{sl}(2)$ we reproduce in modern terms some classical results from the 1940's. When $\mathfrak{g} \simeq \mathrm{sl}(3)$ and sl(2) is a principal subalgebra, bounded simple $(\mathrm{sl}(3), \mathrm{sl}(2))$-modules are Harish-Chandra modules and our result singles out all Harish-Chandra modules with bounded sl(2)-multiplicities. A case where the result is entirely new is the case of a principal sl(2)-subalgebra of $\mathfrak{g} = \mathrm{sp}(4)$.

Keywords: Harish-Chandra modules, bounded sl(2)-multiplicities, sl(2)-characters.

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