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A Quantum Type Deformation of the Cohomology Ring of Flag Manifolds

Let $q_1, \ldots, q_n$ be some variables and consider the ring $K := \mathbb{Z}[q_1, \ldots, q_n]/(\prod_{i=1}^n q_i)$. We show that there exists a K-bilinear product $\star$ on $H^*(F_n; \mathbb{Z}) \otimes K$ which is uniquely determined by some quantum cohomology like properties (most importantly, a degree two relation involving the generators and an analogue of the flatness of the Dubrovin connection). Then we prove that $\star$ satisfies the Frobenius property with respect to the Poincaré pairing of $H^*(F_n; \mathbb{Z})$; this leads immediately to the orthogonality of the corresponding Schubert type polynomials. We also note that if we pick $k \in \{1, \ldots, n\}$ and we formally replace q_k by 0, the ring $(H^*(F_n; \mathbb{Z}) \otimes K, \star)$ becomes isomorphic to the usual small quantum cohomology ring of F_n, by an isomorphism which is described precisely.

Keywords: Flag manifolds, cohomology, quantum cohomology, periodic Toda lattice, Schubert polynomials.

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