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Invariant Berezin Integration on Homogeneous Supermanifolds

Let $\mathcal{G}$ be a Lie supergroup and $\mathcal{H}$ a closed subsupergroup. We study the unimodularity of the homogeneous supermanifold $\mathcal{G}/\mathcal{H}$, i. e. the existence of $\mathcal{G}$-invariant sections of its Berezinian line bundle. To that end, we express this line bundle as a $\mathcal{G}$-equivariant associated bundle of the principal $\mathcal{H}$-bundle $\mathcal{G} \to \mathcal{G}/\mathcal{H}$. We also study the fibre integration of Berezinians on oriented fibre bundles. As an application, we prove a formula of 'Fubini' type:

$$\int_{\mathcal{G}} f = (-1)^{\dim \mathfrak{h}_1 \cdot \dim \mathfrak{g}/\mathfrak{h}} \int_{\mathcal{G}/\mathcal{H}} \int_{\mathcal{H}} f, \text{ for all } f \in \Gamma_c(G, \mathcal{O}_{\mathcal{G}}).$$

Moreover, we derive analogues of integral formulae for the transformation under local isomorphisms $\mathcal{G}/\mathcal{H} \to \mathcal{S}/\mathcal{T}$, and under the products of Lie subsupergroups $\mathcal{M} \cdot \mathcal{H} \subset \mathcal{U}$. The classical counterparts of these formulae have numerous applications in harmonic analysis.

Keywords: Supermanifold, Lie supergroup, homogeneous superspace, Berezin integral, invariant Berezinian form, unimodularity, Fubini formula, fibre integration.

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