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Invariant Polynomials for Multiplicity Free Actions

This work concerns linear multiplicity free actions of the complex groups $G_{\mathbb{C}} = GL(n, \mathbb{C})$, $GL(n, \mathbb{C}) \times GL(n, \mathbb{C})$ and $GL(2n, \mathbb{C})$ on the vector spaces $V = Sym(n, \mathbb{C})$, $M_n(\mathbb{C})$ and $Skew(2n, \mathbb{C})$. We relate the canonical invariants in $\mathbb{C}[V \oplus V^*]$ to spherical functions for Riemannian symmetric pairs (G, K) where $G = GL(n, \mathbb{R})$, $GL(n, \mathbb{C})$ or $GL(n, \mathbb{H})$ respectively. These in turn can be expressed using three families of classical zonal polynomials. We use this fact to derive a combinatorial algorithm for the generalized binomial coefficients in each case. Many of these results were obtained previously by Knop and Sahi using different methods.

Keywords: Multiplicity free actions, invariant theory, symmetric functions.

MSC: 20G05, 13A50; 05E15