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Examples of Self-Iterating Lie Algebras, 2

We study properties of self-iterating Lie algebras in positive characteristic. Let $R = K[t_i | i \in \mathbb{N}]/(t_i^p | i \in \mathbb{N})$ be the truncated polynomial ring. Let $\partial_i = \frac{\partial}{\partial t_i}$, $i \in \mathbb{N}$, denote the respective derivations. Consider the operators

$$v_1 = \partial_1 + t_0(\partial_2 + t_1(\partial_3 + t_2(\partial_4 + t_3(\partial_5 + t_4(\partial_6 + \cdots))))); \qquad (1)$$

$$v_2 = \qquad \partial_2 + t_1(\partial_3 + t_2(\partial_4 + t_3(\partial_5 + t_4(\partial_6 + \cdots)))). \qquad (2)$$

Let $\mathbf{L} = \mathrm{Lie}_p(v_1, v_2) \subset \mathrm{Der}\, R$ be the restricted Lie algebra generated by these derivations. We establish the following properties of this algebra in case $p = 2, 3$.
a) $\mathbf{L}$ has a polynomial growth with Gelfand-Kirillov dimension $\ln p / \ln((1+\sqrt{5})/2)$.
b) the associative envelope $\mathbf{A} = \mathrm{Alg}(v_1, v_2)$ of $\mathbf{L}$ has Gelfand-Kirillov dimension $2 \ln p / \ln((1+\sqrt{5})/2)$.
c) $\mathbf{L}$ has a nil-p-mapping.
d) $\mathbf{L}$, $\mathbf{A}$ and the augmentation ideal of the restricted enveloping algebra $\mathbf{u} = u_0(\mathbf{L})$ are direct sums of two locally nilpotent subalgebras. The question whether $\mathbf{u}$ is a nil-algebra remains open.
e) the restricted enveloping algebra $u(\mathbf{L})$ is of intermediate growth. These properties resemble those of Grigorchuk and Gupta-Sidki groups.

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